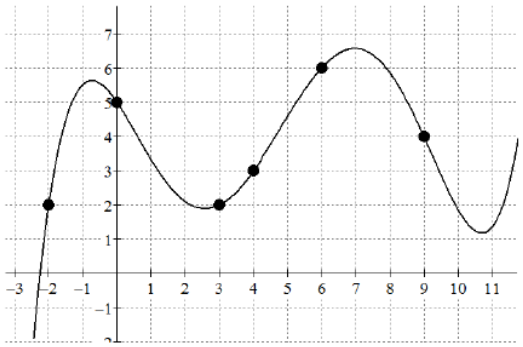
AP CALCULUS BC
Unit 7 Outline – Applications of Integration

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/6	**ESTIMATING AREA USING RIEMANN SUMS**	**Ex. 1** Using the graph of $f(x)$, approximate each definite integral using the indicated method.



Definite Integral	Method	Approximation
$\int_3^9 f(x)dx$	$LRAM_3$	
$\int_0^9 f(x)dx$	$RRAM_4$	
$\int_{-2}^4 f(x)dx$	$TRAP_3$	

Ex. 2 Approximate the area under $f(x)=x^2+1$ on the interval $[0,2]$ using 4 subintervals of equal width.

a) $LRAM_4$
b) $RRAM_4$
c) $MRAM_4$
d) $TRAP_4$

For (a), (b), and (d), explain if each approximation is an overestimate or an underestimate. Give a reason for your answer.

AP MULTIPLE CHOICE

x	0	2	4	6
$f(x)$	4	k	8	12

The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

HOMEWORK	Worksheet 49

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/7	**DEFINITE INTEGRAL AS A RIEMANN SUM**	**Rewrite the definite integral as a limit of a Riemann Sum** **Ex. 1** $\int_0^{\pi} \sin x dx$ **Ex. 2** $\int_2^6 \frac{1}{5}x^2 dx$ **Ex. 3** $\int_1^e \ln x dx$ **Rewrite the limit of a Riemann sum as a definite integral** **Ex. 4** $\lim_{n\to\infty}\sum_{i=1}^{n}\ln\left(2+\frac{5i}{n}\right)\cdot\frac{5}{n}$ **Ex. 5** $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{4+\frac{2i}{n}}\cdot\frac{2}{n}$ **Ex. 6** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{12k}{n}\cos\left(1+\frac{4k}{n}\right)\cdot\frac{4}{n}$ **Ex. 7** $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{10i}{n}\left(\sqrt{1+\frac{5i}{n}}\right)\cdot\frac{5}{n}$

AP MULTIPLE CHOICE

The definite integral $\int_0^4 \sqrt{x}\ dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}\ dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}\ dx < R$

(C) $R < \int_0^4 \sqrt{x}\ dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}\ dx < L$

HOMEWORK	Worksheet 50

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/8	**THE ACCUMULATION FUNCTION AND ITS DERIVATIVE (FTC 2)**	Find the derivative of each function: **Ex. 1** $g(x)=\int_{\sin x}^{\cos x}\ln t dt$ **Ex. 2** $g(x)=\int_{x^3}^{e^x}\tan t dt$ **Notes Handout**
HOMEWORK		Worksheet 51

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/12	**INTERPRETING BEHAVIOR OF ACCUMULATION FUNCTIONS INVOLVING AREA**	**Notes Handout**
HOMEWORK		Worksheet 52

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/13	**AVERAGE VALUE OF A FUNCTION**	**Ex. 1** Find the average value of $f(x) = 5x^4 + 3x^2$ on the interval $-1 \le x \le 2$. **Ex. 2** Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$. **Ex. 3** Find the average value of $f(x) = \frac{1}{x}$ on the interval $[e, 2e]$. **Ex. 4** Find the average value of $y = 3x^2 + 2x$ on the interval $[-1, 2]$
AP MULTIPLE CHOICE The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute? (A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833		
HOMEWORK		Worksheet 53

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/14	**PARTICLE MOTION USING DEFINITE INTEGRALS**	**Notes Handout**
AP MULTIPLE CHOICE A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = \sin(2t)$. If the position of the particle at time $t = \frac{\pi}{2}$ is $x = 4$, what is the particle's position at time $t = 0$? (A) $-\frac{1}{2}$ (B) 2 (C) 3 (D) 5 (E) 8		
HOMEWORK		Worksheet 54

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/15	**QUIZ**	Riemann Sums, Average Value, Accumulation Functions
HOMEWORK		Have a Happy Thanksgiving!!

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/18	**INTEGRAL AS A NET CHANGE** **RATES OF CHANGE**	**Notes Handout**

AP MULTIPLE CHOICE

A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

HOMEWORK	Worksheet 55

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/19	**INTEGRAL AS A NET CHANGE** **RATES OF CHANGE**	AP – Free Response Questions
HOMEWORK		Worksheet 56

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/20	**AREA BETWEEN TWO CURVES**	**Examples from Homework #1, 5, and 9**

AP MULTIPLE CHOICE

What is the area of the region enclosed by the graphs of $y = \frac{1}{1+x^2}$ and $y = x^2 - \frac{1}{3}$?

(A) 0.786 (B) 0.791 (C) 1.582 (D) 1.837 (E) 1.862

HOMEWORK	Worksheet 57

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/21	VOLUME (KNOWN CROSS SECTION)	**Notes Handout**

AP MULTIPLE CHOICE

Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3 - x)$, for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.442 (B) 1.029 (C) 1.296 (D) 3.233 (E) 4.071

HOMEWORK	Worksheet 58

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/22	VOLUME OF SOLIDS FORMED BY ROTATION	**Notes - Handout**

AP MULTIPLE CHOICE

A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115

HOMEWORK	Worksheet 59

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
12/2	VOLUME OF SOLIDS FORMED BY ROTATION THE SHELL METHOD	**Notes - Handout**

HOMEWORK	Worksheet 60

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
12/3	**ARC LENGTH**	**Notes - Handout**

AP MULTIPLE CHOICE

The length of the curve $y = x^4$ from $x = 1$ to $x = 5$ is given by

(A) $\int_1^5 \sqrt{1 + 4x^3}\, dx$ (B) $\int_1^5 \sqrt{1 + x^4}\, dx$ (C) $\int_1^5 \sqrt{1 + 4x^6}\, dx$ (D) $\int_1^5 \sqrt{1 + 16x^6}\, dx$ (E) $\int_1^5 \sqrt{1 + x^8}\, dx$

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	–1

The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6 (B) 34 (C) $2\sqrt{3} + 2\sqrt{2} + 2$ (D) $2\sqrt{5} + 2\sqrt{2} + 2$ (E) $2\sqrt{5} + 4\sqrt{2} + 2$

HOMEWORK	Worksheet 61

12/6 – Review

12/7 – Unit Exam